Stochastic Processes and Thermodynamics on Curved Spaces

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Abstract

Our approach views the thermodynamics and kinetics in general relativity and extended gravitational theories (with generic local anisotropy) from the perspective of the theory of stochastic differential equations on curved spaces. Nonequilibrium and irreversible processes in black hole thermodynamics are considered. The paper summarizes the author's contribution to Journees Relativistes 99 (12–17 September 1999), Weimar, Germany.

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1 Diffusion, Kinetics and Thermodynamics in Locally Isotropic and Anisotropic spacetimes

We generalized [8] the stochastic calculus on Riemannian manifolds for anisotropic processes anf for fiber bundles provided with nonlinear connection structure [4]. Lifts in the total space of linear frame bundles were used in order to consider Browinian motions, Wiener processes and Langevin equations in a covariant fashion. The concept of thermodynamic Markovicity and Chapman–Kolmogorov equations were analyzed in connection to the possibility of obtaining information about pair–correlation functions on curved spaces.

Fokker–Plank type covariant equations were derived for both locally isotropic and anisotropic gravitational and matter field interactions. Stability of equilibrium and nonequilibrium states, evolution criteria, fluctuations and dissipation are examinded from the view point of a general stochastic formalism on curved spaces.

The interrelation between classical statistical mechanics, thermodynamics and kinetic theory (the Bogolyubov — Born and Green — Kirkwood — Yvon herachy, and derivation of Vlasov and Boltzmann equations [10]) was studied on Riemannian manifolds and vector bundles.

The covariant diffusion and hydrodynamical approximations [3], the kinematics of relativistic processes, transfering and production of entropy, dynamical equations and thermodynamic relations were consequently defined. Relativistic formulations [1] and anisotropic generalizations were considered for extended irreversible thermodynamics.

2 Thermodynamics of Black Holes with Local Anisotropy

The formalism outlined in the previous section was applied to cosmological models and black holes with local spacetime anisotropy [6, 7].

We analyzed the conditions when the Einstein equations with cosmological constant and matter (in general relativity and low dimenensional and extended variants of gravity) describe generic locally anisotropic (la) spacetimes. Following De Witt approach we set up a method for deriving energy momentum tensors for locally anisotropic matter.

We speculated on black la–hole solutions induced by locally anisotropic splittings from tetradic, spinor and gauge and generalized Kaluza–Klein–Finsler models of gravity [4, 9]. Possible extensions of la–metrics [5, 6] to string and brane models were considered.

The thermodynamics of (2+1) dimensional black la–holes was discussed in connection with a possible statistical mechanics background based on locally anisotropic variants of Chern–Simons theories [7]. We proposed a variant of

irreversible thermodynamics for black la–holes. There were also considered constructions and calculus of thermodynamic parameters of black la–holes, in the framework of approaches to thermodynamic geometry [2] for nearly equilibrium states, and the effects of local nonequilibrium and questons of stability were analyzed by using thermodynamic metrics and curvatures.

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